

# Wireless Scheduling

Mung Chiang

Electrical Engineering, Princeton University

WiOpt

Seoul, Korea

June 23, 2009

## Outline

- Structured teaser on wireless scheduling
- Focus on key ideas and 10 open problems
- Biased highlights on 3D tradeoff and CSMA
- Optimization combined with applied probability
  
- **Acknowledgement:** coauthors of papers cited in the talk:  
Rob Calderbank, Jang-Won Lee, Jiaping Liu, Vince Poor, Alexandre Proutiere, Yung Yi, Junshan Zhang  
Book chapter on the subject with Yung Yi
- **Apology:** for missing references and unbalanced emphasis

## The Central Question

In an interference environment, who can talk in each time slot?

## The Basic Problem Statement

**Given:** Who can interfere with whom

- Topology  $G = (V, L)$
- Model and representation (graph, set, matrix) of interference

**Variables:** Who talks when

- Activation vector  $s$ , Contention probability  $p, \lambda$ , Holding time  $\mu$

**Goal:** Stable, Fair, Small delay, Big utility

**Stochastic optimization:** Workload arrival, Algorithm, (Channel)

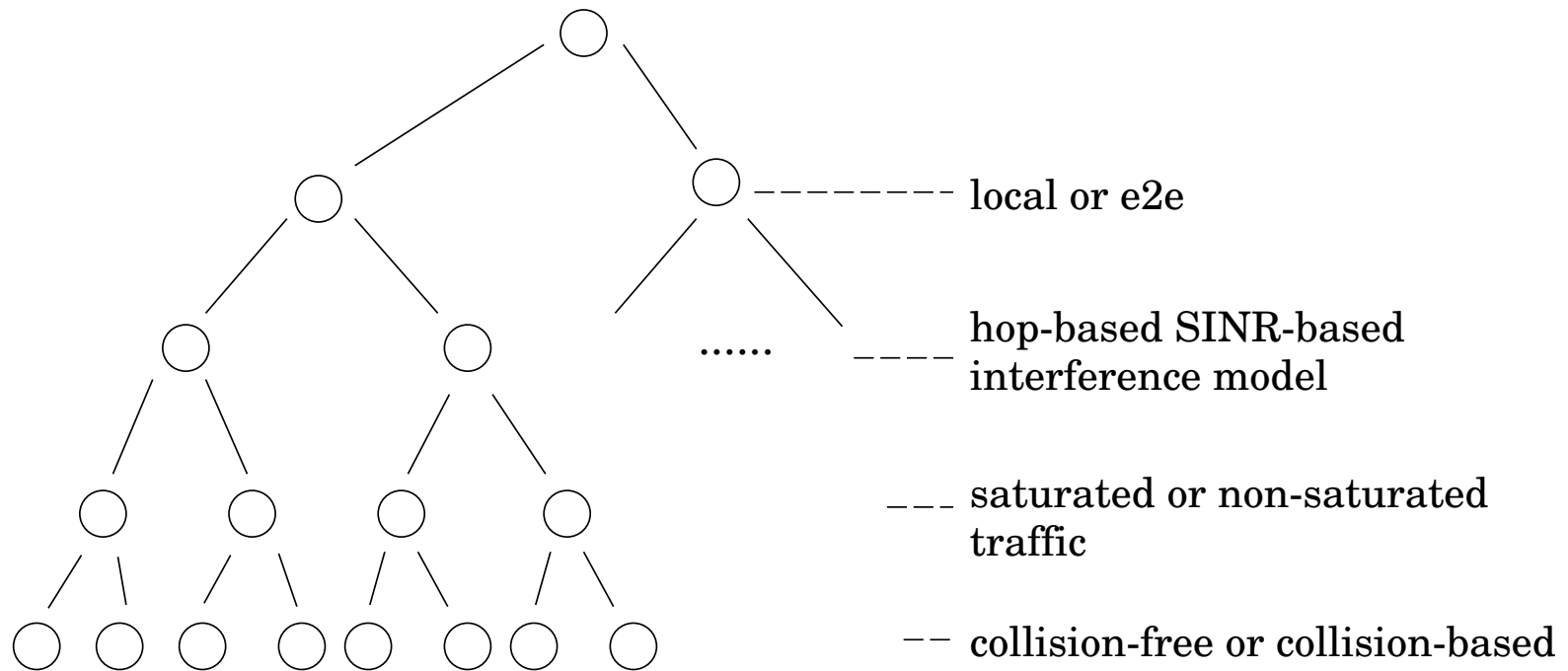
## Practice-Theory Dichotomy

**Simple** ones used, analysis can be very challenging:

- Aloha
- CSMA/CA, CSMA/CD
- RTS/CTS

**Sophisticated** algorithms based on graph, optimization, game theories

## Tree of Problems



## Taxonomy of Problems

- Local contention neighborhood
- End-to-end (with routing and rate control)
  
- $K$ -hop interference model ( $K = 1$  bluetooth,  $K = 2$  802.11)
- SIR-based interference model (and adaptive physical layer)
  
- Saturated traffic (utility, fairness)
- Non-saturated (stability region, delay)
  
- Contention-free
- Contention-based

## End-to-End

- Unsaturated

Joint congestion control, routing, and scheduling: Lin Shroff 2005, Neely Modiano Li 2005, Eryilmaz Srikant 2005, Stolyar 2005, Chen Low Chiang Doyle 2006...

- Saturated

Joint congestion control and contention control: Wang Kar 2005, Lee Chiang Calderbank 2006, Zhang Zheng Chiang 2007...

- Combination

Bui Eryilmaz Srikant Wu 2006, Chaporkar Sarkar 2006, Eryilmaz Ozdaglar Modiano 2007, Sharma Shroff Mazumdar 2007...

## End-to-End

Joint congestion control, routing, and scheduling:

- Link based formulation
- Node based formulation: per-destination queues, includes routing

$$x_i^k \leq \sum f_{ij}^k - \sum f_{ji}^k \rightarrow \max_f \sum_{ij} f_{ij} \max_k (q_i^k - q_j^k)$$

Combination of **backpressure** and **congestion pricing**

**Bottleneck** is scheduling

**More subtle points:**

- Architectural choices: Layering as Optimization Decomposition
- Dual variable not exactly the queue size

## SIR Based Interference Model

- **Limited work** on limited models:

Cruz Santhanam 2003

Johansson Xiao 2006

Yi de Veciana Shakkottai 2007

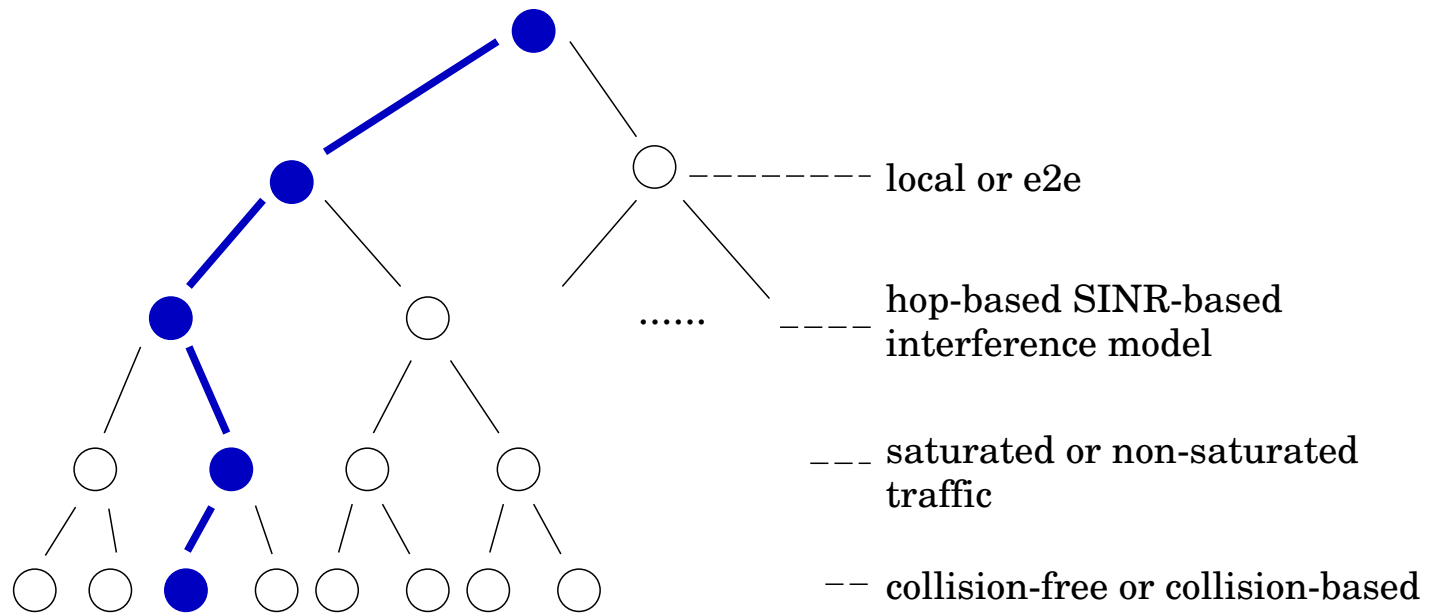
Kompella Wieselthier Ephremides 2008

High SIR models...

Further complications:

- Variable transmit power
- Channel probing
- Capture effect
- Sophisticated decoders

## Where We Are In The Tree



## Maximum Weight

- Tassiulas Ephremides 1992

The max-weight algorithm is choosing the  $s^*(t)$  at each slot  $t$ :

$$s^*(t) = \arg \max_{s \in \mathcal{S}} W(s), \quad W(s) \triangleq \sum_{l \in L} Q_l(t) s_l.$$

$\mathcal{S}$ : Set of feasible schedules

$Q_l(t)$ : Queue size on link  $l$  at time  $t$

Throughput-optimal, Maximum stability region

- Connections to:

Prior work: Hajek Sasaki 1988 (known arrivals)

Graph theory: NP-hard Maximum Weighted Independent Set

Switching theory

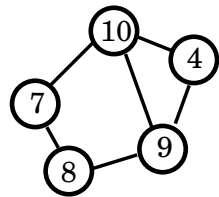
- General yet **complex**. How to make it simple and distributed?

## Approximation: Maximal Weight

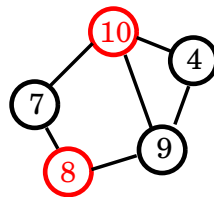
Suboptimal matching that can't be increased by activating more links:

- **Greedy**: the link  $l$  with the largest queue length
- **Locally-greedy**: a random link  $l$  with a locally-longest queue length

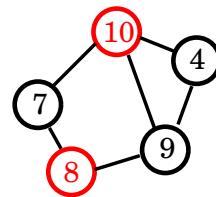
Link  
Conflict Graph



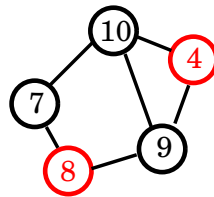
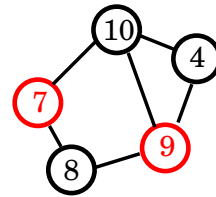
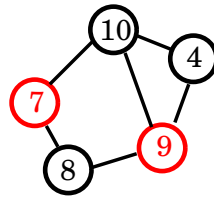
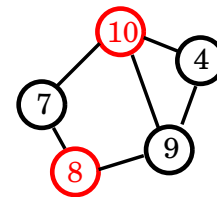
Maximal



Locally  
greedy



Greedy



## Approximation: Maximal Weight

$\gamma = 1/2$  ( $K = 1$ ): Chaporkar Kar Sarkar 2006, Wu Srikant 2006

$2/3$  ( $K = 1$ , tree): Sarkar Kar 2006

1: NP-hard in general ( $K > 1$ ): Sharma, Mazumdar, Shroff 2006

$1/(\text{maximum interference degree})$  Wu Srikant Perkins 2007, Chaporkar Kar Sarkar 2007:  $1/8$  for geometric graph

Further approx: Gupta Lin Srikant 2007

1 under **local pooling condition** (tree): Dimaki Walrand 2006, Brzesinski Zussman Modiano 2006, Zussman Brzezinski Modiano 2008, Joo Lin Shroff 2008:  $1/6$  for 2D geometric graph

Distributed: Israeli Itai 1986, Heopman 2004

## Open Problem

Q1: Lower and upper bounds on throughput by maximal weight scheduling for **general topology and  $K$** ? (Also for the next two parts of the talk)

## Randomization: Pick and Compare

- **Centralized:** Tassiulas 1998

At each time slot  $t$ , the  $\gamma$ -RPC first generates a random schedule  $s'(t)$  satisfying **P**, and then schedule  $s(t)$  defined in **C**:

**P**  $\exists 0 < \delta \leq 1$ , s.t.  $\text{Prob}(s'(t) = s | Q(t)) \geq \delta$ , for some schedule  $s$ , where  $W(s) \geq \gamma W^*(t)$

**C**  $s(t) = \arg \max_{s \in \{s(t-1), s'(t)\}} W(s)$

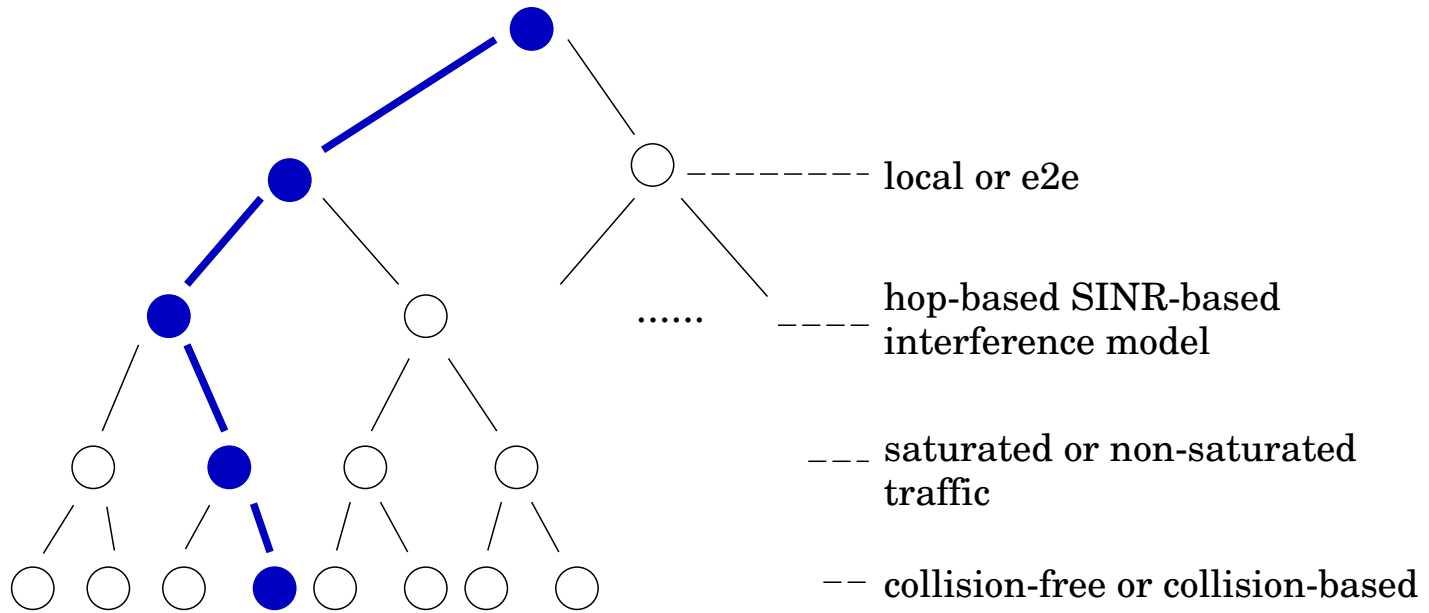
- **Message passing** with gossip: P and C can be inaccurate

$\gamma = 1$  ( $K = 1$ , not counting complexities): Modiano Shah Zussman 2006

## Open Problem

Q2: What's the largest **effective throughput** of RPC for general  $K$ ?

## Where We Are In The Tree



## Message Passing Random Access

$K = 1$ . Each slot starts with constant  $M$  minislots for control signals

- Compute  $0 \leq x_l(t) \leq 1$  using queue lengths of the interfering neighbors via message passing:

$$x_l(t) = \frac{Q_l(t)}{\max \left[ \sum_{k \in L(t(l))} Q_k(t), \sum_{k \in L(r(l))} Q_k(t) \right]}, \quad (1)$$

- The link  $l$  contends each mini-slot with the probability  $p_l = f(x_l(t), M)$  for some  $f$  (e.g.,  $g(M)x/M, 1 - \exp(-g(M)x/M)$ )
- Successfully contended link transmits during the time slot

$1/3 - 1/M$ : Lin Rasool 2006

$1/2 - 1/\sqrt{M}$ : Joo Shroff 2007

$1/2 - \log(2M)/2M$ : Gupta Lin Srikant 2007

Further study: Marbach Eryilmaz Ozdaglar 2007, Joo Lin Shroff 2008

## Pause

What about the **overhead**? Computation and Communication

## Detour: Distributed Algorithm in Networking

### How distributed is distributed?

Dimensions to quantify explicit message passing:

- How often? Time-complexity
- How far? Space-complexity
- How many bits per message? Bit-complexity

### Performance-Distributeness tradeoff:

- Outer bound for benchmarking
- Inner bound by protocol design
- Design ideas and proof techniques

## Detour: Optimization Without Optimality

- **Optimality-driven design:**

Under the constraint of having an optimality proof, find the simplest protocol

- **Simplicity-driven design:**

Under the constraint of zero message passing, find the best performance protocol

Expand the conditions of convergence, optimality...

Bound the optimality gap, stability region reduction...

- **Overhead changes the accounting rule:**

Multiplier effect

Sweet spots in the tradeoff

## Throughput-Complexity Tradeoff

Local versions of RPC:

[Graph partitioning](#): Ray Sarkar 2007

[Link augmentation](#): Sanghavi Bui Srikant 2007

Extension to general  $K$ : Jung Shah 2007, Yi Chiang 2008

## Throughput-Delay-Complexity Tradeoff

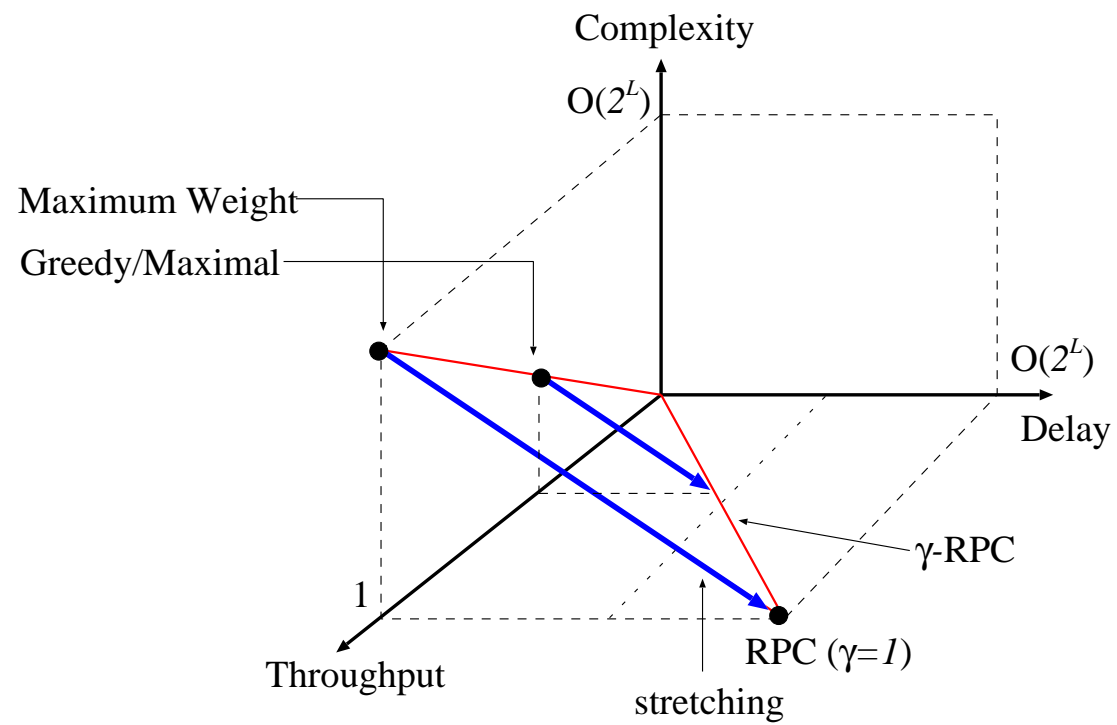
Parameterization:  $(\gamma, \xi, \chi)$  approximate algorithm

Stretching by  $m$ : stability unaffected, delay grows linearly in  $m$

- From  $(\gamma, \xi, \chi)$  to  $(\gamma, \xi + mV\Omega(1 + \gamma), \chi/m)$
- Each scheduling algorithm is one point in 3D tradeoff space
- Parameterize into tradeoff curves
- Three 2D projections: e.g., Stability-delay tradeoff for a fixed complexity

Yi Proutiere Chiang 2008

## 3D Tradeoff



## Open Problem

Q3: Only achievable curves. What about **achievability surface** or **converse**?

Q4: Tradeoff with **spatial-complexity** and **bit-complexity** (event-triggered, differential-coded)?

Q5: Only a bound on delay. **Can we understand delay better and minimize delay?** (Tight bounds for various algorithms in general graph and for general  $K$ )

## Delay Characterization

- The challenge of **dimensionality**
- Switching literature sometimes helpful
- **Lyapunov bound:**

Neely 2006, Neely 2008, Chaporkar et al 2008, Gupta Shroff 2009

$$Q(t + 1) = [Q(t) - D(t) + A(t)]^+$$

Upper bound  $\mathcal{O}(\log \max_l N(l))$  Maximal Weight and Markov bursty traffic

Lower bound for multihop backpressure with fixed routing

- **Large deviation:**

Venkataramanan and Lin 2006, Ying Srikant Dullerud 2006

Delay bound violation probability constraint

Related: scheduling under deadline constraints

## Delay Characterization

- Heavy traffic approximation:

Shakkottai Srikant Stolyar 2004, Shah Wischik 2007

Assume heavy traffic regime and diffusion scale  $\hat{x}^n(t) = X(n^2t)/n$

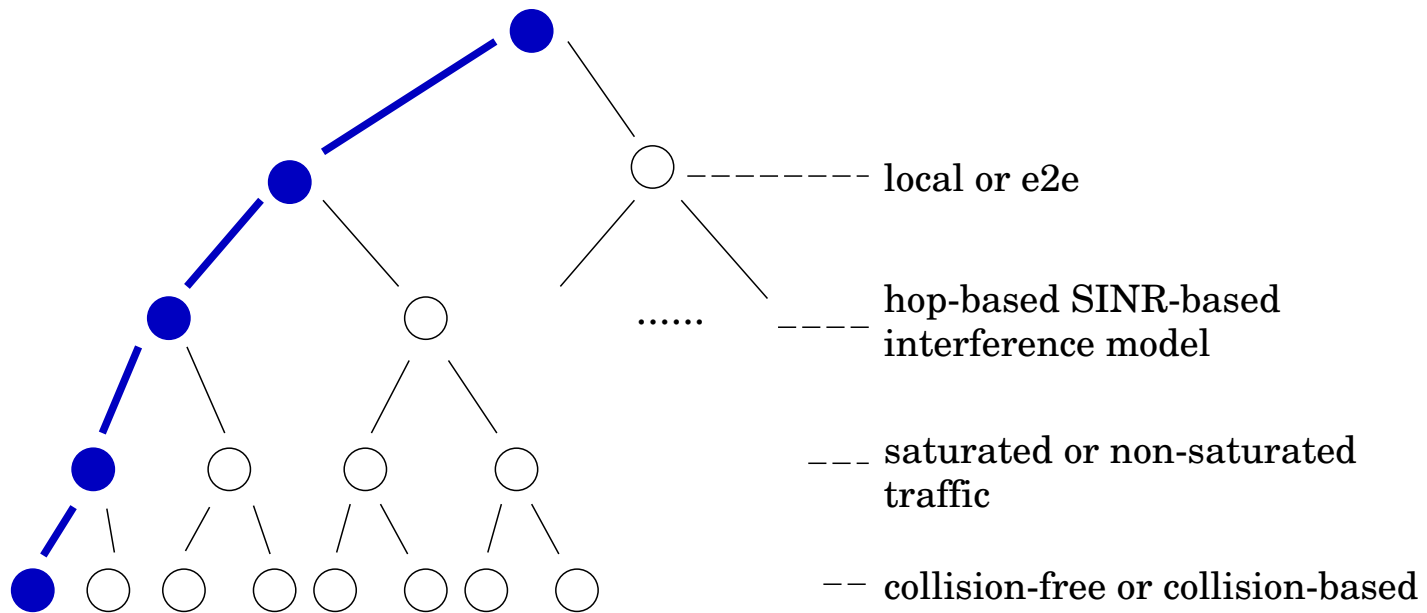
Prove state space collapse and characterize workload process

Derive inference to the original problem

Yi Zhang Chiang 2009

Vacation model for complexity: exponential growth

## Where We Are In The Tree



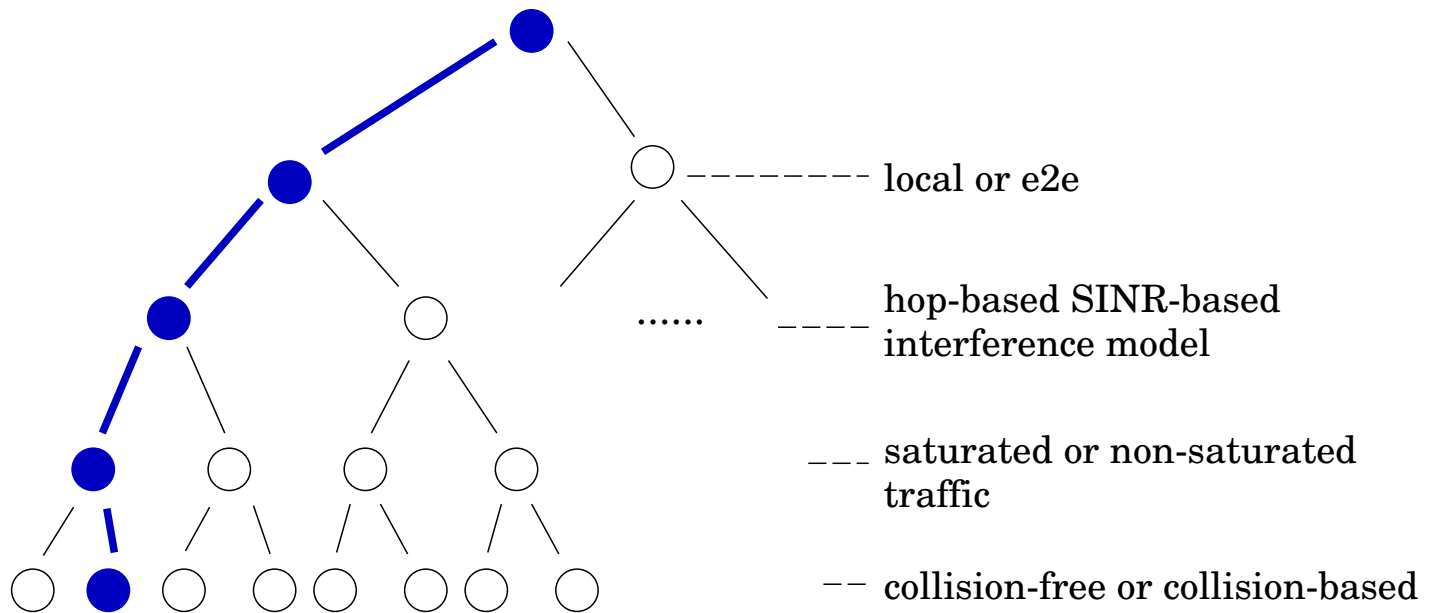
## Contention Graph

Nandagopal Kim Gao Bharghavan 2000

Chen Low Doyle 2005

Turns the problem to one [similar to congestion control](#)

## Where We Are In The Tree



## Contention Probability for Slotted Aloha

Proportional fair: Kar Sarkar Tassiulas 2004

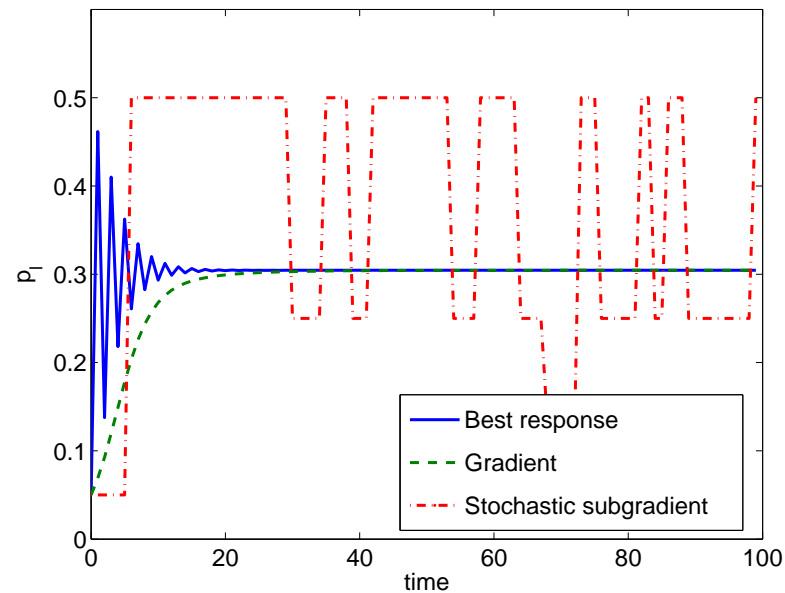
General utility: Lee Chiang Calderbank 2006

Queue backpressure: Gupta Stoylar 2006, Stoylar 2008, Liu Stoylar  
Chiang Poor 2008

## Reverse Engineering Exponential Backoff

- Reverse engineer as a **game** (derive utility function)
- Nash equilibrium exists but suboptimal
- Existing protocol is **stochastic subgradient**
- Converges under conditions on how interfered the topology is

Lee Chiang Calderbank 2007



## Reverse Engineering Exponential Backoff

- **Contrast** to reverse engineering of TCP congestion control into NUM
- Self interests not aligned
- **How to align them?** Maybe with the help of message passing?

## Problem Statement

$L_{out}(n)$ : set of logical links where node  $n$  is transmitter

$N(l)$ : set of nodes whose transmission collide with that on  $l$

Each link with a utility function  $U_l(x_l)$  and fixed rate  $c_l$

$$x_l = c_l p_l \prod_{k \in N(l)} (1 - P^k)$$

Optimization over variables  $(\mathbf{p}, \mathbf{P})$ :

$$\begin{aligned} & \text{maximize} && \sum_l U_l(c_l p_l \prod_{k \in N(l)} (1 - P^k)) \\ & \text{subject to} && x_l^{min} \leq c_l p_l \prod_{k \in N(l)} (1 - P^k) \leq x_l^{max}, \quad \forall l \\ & && \sum_{l \in L_{out}(n)} p_l = P^n, \quad \forall n \\ & && P^{min} \leq P^n \leq P^{max}, \quad \forall n, \quad 0 \leq p_l \leq 1, \quad \forall l \end{aligned}$$

## How Distributed Can Solution Be

- Step 1: log change of variable to decouple
- Step 2: dual decomposition
- Step 3:  $\alpha \geq 1$  utility function to ensure global optimality
- How to make it converge **faster**?

Stepsize-free algorithm

- How to reduce message passing to **zero**?

**Learn** from historical record of collisions

Optimal for fully-interfered topology and sufficient number of nodes

Mohsenian-Rad Huang Chiang Wong 2009

## Open Problem

Q6: How suboptimal is utility maximization by Aloha with no message passing?

## Utility-Optimal CSMA

No message passing (think converse point in 3D tradeoff)

- Utility in saturated case
- Rate stability in non-saturated case

Adaptive CSMA:

- Jiang Walrand 2008
- Rajagopalan Shah 2008
- Liu Yi Proutiere Chiang Poor 2008

Related: Marbach Eryilmaz 2008, Liew et al 2008

Key background: Kelly 1987, Hajek 1988, Borkar 2006

## Problem Statement

- $\boldsymbol{\gamma} = (\gamma_l, l \in \mathcal{L})$ : long-term throughputs
- $\Gamma$ : throughput region

$$\Gamma = \left\{ \boldsymbol{\gamma} : \exists \boldsymbol{\tau} \in \Upsilon, \forall l \in \mathcal{L}, \gamma_l \leq \sum_{s \in \mathcal{S}: s_l=1} \tau_s \right\}$$

where  $\Upsilon = \{ \boldsymbol{\tau} = (\tau_s, s \in \mathcal{S}), \forall s, \tau_s \geq 0, \sum_{s \in \mathcal{S}} \tau_s = 1 \}$

Optimization problem:

$$\max \sum_l U(\gamma_l), \text{ s.t. } \boldsymbol{\gamma} \in \Gamma$$

## Two Timeslot Models

- Poisson clock contention

Mathematically, no collision

More tractable starting point

Turns out optimality can be asymptotically approached arbitrarily tightly

- Discrete time contention and backoff

Represent the reality and incorporate collision

Need to bound both algorithm inefficiency and collision degradation

Can form a sequence of systems converging to Poisson clock model

Throughput gap and efficiency-fairness tradeoff

## Timescale Assumption

Two interacting components:

- Continuous time: defines at each instant which links are transmitting
- Discrete time: periodically updates the CSMA transmission parameters  $(\lambda_t, \mu_t)$  used in the first component

Two timescales:

- Easy: Freeze CSMA parameters over a frame of timeslots, wait for stochastic network state converge to stationary distribution
- Hard: Underlying stochastic network and CSMA transmission parameters evolve simultaneously

## Algorithm

Parameters:  $V > 0$ ,  $W(\cdot)$ ,  $b(t)$

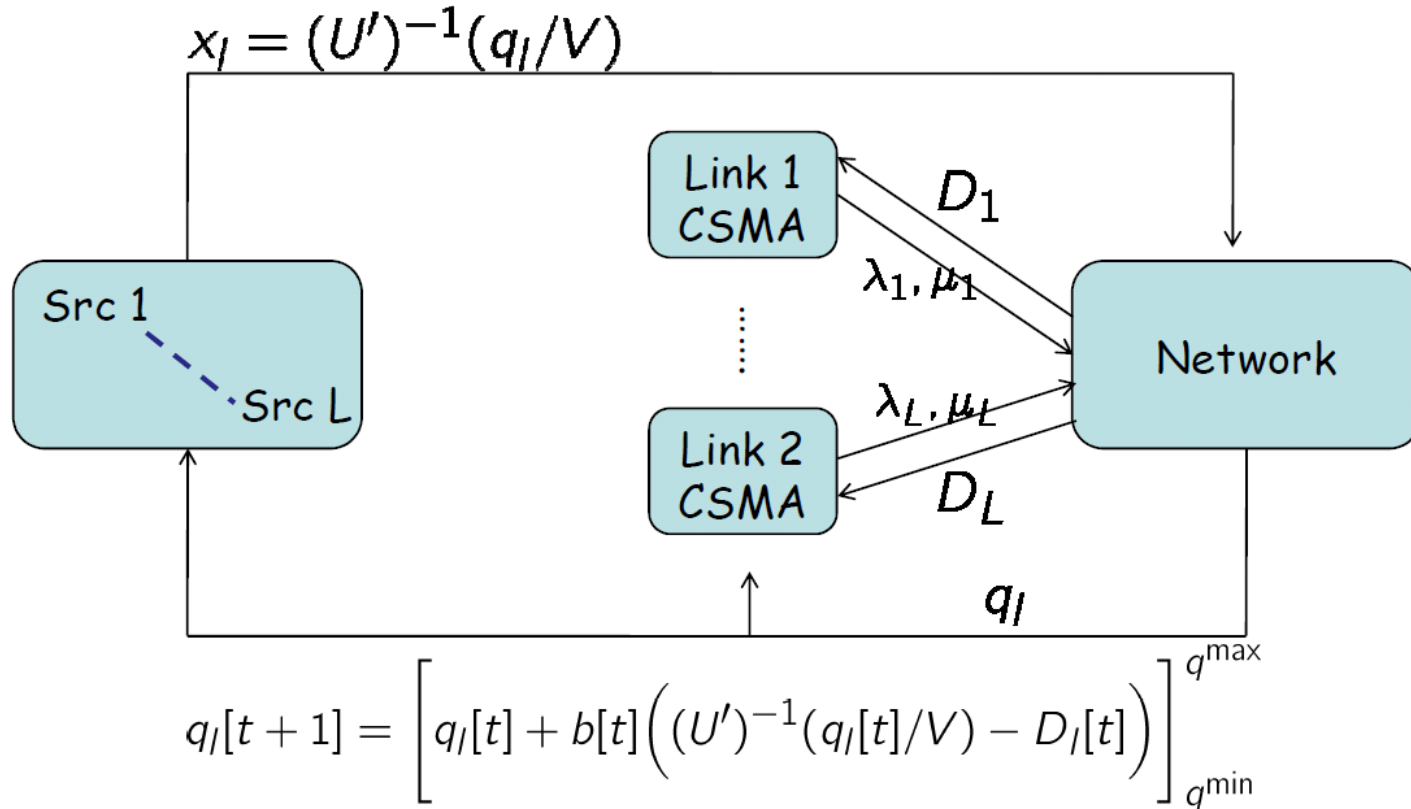
(e.g.,  $V = 10$ ,  $W(x) = \log \log(x + e)$  or  $W(x) = x$ ,  $b(t) = 1/t$ )

$$\bullet q_l[t + 1] = \left[ q_l[t] + \frac{b[t]}{W'(q_l[t])} \left( U_l'^{-1} \left( \frac{W(q_l[t])}{V} \right) - D_l[t] \right) \right]_{q^{\min}}^{q^{\max}},$$

$$\bullet \rho_l[t + 1] = \exp\{W(q_l[t + 1])\}$$

- The corresponding  $\lambda_l[t + 1]$  and  $\mu_l[t + 1]$  updated such that  $\rho = \lambda/\mu$

## Algorithm With Congestion Control



$$\lambda_l[t+1] = \mu^{-1} \exp(q_l[t+1])$$

## Performance

Convergence to:  $\lim_{t \rightarrow \infty} \mathbf{q}[t] = \mathbf{q}^*$

The corresponding throughput  $\boldsymbol{\gamma}(\boldsymbol{\rho}(\mathbf{q}^*))$  solves:

$$\begin{aligned} &\text{maximize} && V \sum_{l \in \mathcal{L}} U(\gamma_l) - \sum_s \tau_s \log \tau_s \\ &\text{subject to} && \gamma_l \leq \sum_{s \in \mathcal{S}: s_l=1} \tau_s \\ &&& \sum_s \tau_s = 1 \end{aligned}$$

Approximately solves utility maximization. **Max error:  $\log |\mathcal{S}|/V$**

As  $V \rightarrow \infty$  with speed  $\mathcal{O}(L)$ , it solves utility maximization

## Proof

- As a stochastic subgradient algorithm modulated by a Markov chain
- Main step 1: show averaging over fast timescale is valid

Interpolation of discrete  $q$  converges a.s. to a continuous  $q$  solving a system of ODE

- Main step 2: show the resulting averaged process converge

The system of ODE describes the trajectory of subgradient to solve the dual problem

- Main step 3: Standard methods in convex optimization and duality

Based on our approach. See also other proofs that modify the algorithm

## Detour: A General Lemma

Given sequence  $x_n$  of random real numbers, and random variable  $Y_n$ ,

$$x_{n+1} = x_n + b_n h(x_n, Y_n)$$

$h$  is bounded, continuous, Lipschitz (to first variable)

$Y_n$  is Markov chain whose kernel evolves in time and depends on  $x_n$ :

$$\text{Prob}[Y_{n+1} = z | Y_n = y, x_n = x] = p(z|y, x)$$

Kernel  $p$  of a stationary, ergodic Markov chain with stationary distribution  $\pi_x$

Let  $\bar{x}$  be interpolated  $x$ , and  $\tilde{x}^s$  be solution to the following ODE:

$$\frac{dx(t)}{dt} = \sum_y \pi_{x(t)}(y) h(x(t), y), \quad \tilde{x}^s(0) = \bar{x}(s)$$

Then, a.s.,

$$\lim_{s \rightarrow \infty} \sup_{t \in [s, s+T]} |\bar{x}(t) - \tilde{x}^s(t)| = 0$$

See Borkar proof and Proutiere proof

## Discrete Slot Model: Efficiency-Fairness Tradeoff

Contention probability:  $p_l = \epsilon \lambda_l$ . Channel holding  $1/\epsilon \mu_l$

Average number of periods during which link  $l$  do not transmit successfully:  $E_l = \frac{1}{\epsilon \mu} \times \frac{1 - \gamma_l(\rho^*)}{\gamma_l(\rho^*)}$

Short-term fairness index:  $\beta = 1 / \max_l E_l$  (worst transient delay)

Contrast to long-term fairness (equilibrium throughput utility)

For fully-interfered network, to guarantee a loss of utility of  $\delta$ ,

$$\text{without RTS/CTS: } \beta \leq \frac{\delta}{C_1 \exp(C_2/\delta)},$$

$$\text{with RTS/CTS: } \beta \leq \frac{\delta}{C_3}.$$

Based on our approach. See also Ni Srikant 2009

## Open Problem

Q7: **3D tradeoff** and **transient behavior** of utility-optimal adaptive CSMA?

Q8: **Queue stability** for non-saturated arrival: can CSMA with zero message passing be optimal?

Q9: Implementation and **deployment** of utility optimal CSMA?

## Open Problem

Q10: Is it better to control **when** to talk or **how loud** to talk?

Centralized: when to convexify power controlled throughput region?

Distribute: even harder

## Final Thoughts

- Wireless scheduling is **hard**, even for simple models:

High dimensionality and queueing dynamics

Non-convexity and computation complexity

Coupling and communication complexity

- New tools and results are making fast **progress**

Form an intellectual heritage with clear open problems

Need to demonstrate impact in commercial design

## Contacts

[chiangm@princeton.edu](mailto:chiangm@princeton.edu)  
[www.princeton.edu/~chiangm](http://www.princeton.edu/~chiangm)